subject to the boundary conditions:

$$v(x,0) = 0 \quad -x_1 \leqslant x \leqslant x_1 \tag{2A}$$

$$\frac{\partial v}{\partial n} = 0: \quad (x^2 + y^2)^{\frac{1}{2}} = R: \quad y \geqslant 0. \tag{3A}$$

Letting $z = x + iy$, and introducing coaxal coordinates

$$\zeta = \xi + i\eta = \frac{z + x_1}{z - x_1} \tag{4A}$$

we find that

$$\nabla^2 v = 4\frac{\partial^2 v}{\partial z \partial \bar{z}} = \frac{4x_1^2}{|\sinh^2 \zeta|^2}\frac{\partial^2 v}{\partial \zeta \partial \bar{\zeta}} = \frac{-g}{\nu} \tag{5A}$$

subject to

$$\frac{\partial v}{\partial \xi} = 0: \quad \xi = \pi - \theta_0 \tag{6A}$$

$$v = 0: \quad \xi = \pi.$$

Solution of this problem can be effected numerically, or analytically in terms of a complete set of orthogonal functions.

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THE DISPERSION OF MATTER IN TURBULENT SHEAR FLOW

KEMAL M. ATESMEN

Department of Civil Engineering, Colorado State University, Fort Collins, Colorado, U.S.A.

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INTRODUCTION

In the past fifteen years considerable progress has been recorded by researchers concerning the dispersion of matter in turbulent shear flows. The practical applications of this literature to flow metering in pipes and natural streams, as well as pollution control studies and other engineering problems, are numerous. The formulation of the problem involves seeking solutions for specific initial conditions of the transient diffusion equation. The analyses applied to fully developed channel and pipe flows are based on eddy diffusivity approximations using Reynolds' analogy and semi-empirical solutions for the mean flow field.

The purpose of this paper is to present a general method for finding solutions to the longitudinal dispersion problem. A specific example is presented in detail where both experimental data and an earlier analysis are available for comparison. It is concluded that the analytical methodology proposed here not only has a suitable mathematical formalism, but that it also provides a finite algorithm for numerical solution with advantages over previous methods.

PRESENT ANALYSIS

Aris [1] published a method for solving longitudinal dispersion problems which has been successfully applied by several authors in the last three years. Consider a fully developed channel flow of an incompressible fluid where the time averaged concentration C of the dispersing matter may be described by

$$\frac{\partial C}{\partial t} + U\frac{\partial C}{\partial x} = \varepsilon_x\frac{\partial^2 C}{\partial x^2} + \frac{\partial}{\partial y}\left(\varepsilon_y\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(\varepsilon_z\frac{\partial C}{\partial z}\right) \tag{1}$$

where t is the dispersion time, x, y, z, are the space coordinates in the longitudinal, vertical and lateral directions, respectively, and ε is the eddy diffusivity for mass transfer.

To facilitate comparisons, the notation and dimensionless parameters introduced by Aris [1] will be incorporated into equation (1). Following Aris, define the local velocity as

$$U(y,z) \equiv \bar{U}[1 + \chi(y,z)] \tag{2}$$

where $\chi(y,z)$ is a function which describes the variation of velocity in the cross section, and the local eddy diffusivity as

$$\varepsilon_x = \varepsilon_y = \varepsilon_z \equiv D\psi(y,z) \tag{3}$$

where D is the average value of the eddy diffusivity in the cross section and $\psi(y,z)$ is a function describing the distribution of eddy diffusivity.* Using these definitions, equation (1) becomes

* The present analysis assumes isotropic turbulence structure, but the extension to non-isotropic turbulent flows could be achieved by redefining local eddy diffusivities, namely $\varepsilon_x = D_x\psi_x(y,z)$, $\varepsilon_y = D_y\psi_y(y,z)$ and $\varepsilon_z = D_z\psi_z(y,z)$.

$$\frac{\partial C}{\partial t} + \bar{U}(1+\chi)\frac{\partial C}{\partial x} = D\psi\frac{\partial^2 C}{\partial x^2} + D\frac{\partial}{\partial y}\left(\psi\frac{\partial C}{\partial y}\right) + D\frac{\partial}{\partial z}\left(\psi\frac{\partial C}{\partial z}\right) \quad (4)$$

with introduction of the dimensionless parameters

$$\xi = \frac{x - \bar{U}t}{y_n}, \quad \eta = y/y_n, \quad \zeta = z/z_n, \quad \tau = Dt/y_n^2, \quad \mu\bar{U}y_n/D$$

where y_n is the depth of uniform flow in an open channel, equation (4) becomes

$$\frac{\partial C}{\partial t} + \mu\chi\frac{\partial C}{\partial \xi} = \psi\frac{\partial^2 C}{\partial \xi^2} + \frac{\partial}{\partial \eta}\left(\psi\frac{\partial C}{\partial \eta}\right) + \frac{\partial}{\partial \zeta}\left(\psi\frac{\partial C}{\partial \zeta}\right). \quad (6)$$

The governing equation (6) is an Eulerian description of motion and the dispersion is occurring in a frame of reference which is moving with mean velocity. The initial and the boundary conditions for equation (6) are: (1) at $\tau = 0$, $C(\xi, \eta, \zeta, 0)$, the initial spatial distribution of dispersant that is contained in a finite length of channel, is known. The reflection boundary conditions can be stated as (2) $\psi\partial C/\partial n = 0$ at all boundaries including the water surface, where n is the normal to the boundary. That is, no diffusion across the boundaries is permitted. Also the total amount of dispersant in the system remains constant or

$$(3)\frac{1}{A}\iint_A \int_{-\infty}^{\infty} C(\xi, \eta, \zeta, \tau)\,d\xi\,d\eta\,d\zeta = \text{constant},$$

where A is the cross-sectional area of the channel. We can separate the independent variable τ from equation (6) by letting

$$C(\xi, \eta, \tau) = \phi(\xi, \eta)\,T(\tau) \quad (7)$$

and restricting the analysis to two-dimensional flow by setting $\partial/\partial\zeta$ term equal to zero. The resulting equations are

$$\frac{\partial T}{d\tau} + \lambda^2 T = 0 \quad (8)$$

and

$$\psi(\eta)\left(\frac{\partial^2\phi}{\partial\xi^2} + \frac{\partial^2\phi}{\partial\eta^2}\right) + \frac{\partial\psi}{\partial\eta}\frac{\partial\phi}{\partial\eta} - \mu\chi(\eta)\frac{\partial\phi}{\partial\xi} + \lambda^2\phi = 0. \quad (9)$$

Where the λ are the eigenvalues. Equation (8) has an exact solution of the type

$$T_\lambda = H_{1\lambda}e^{-\lambda^2\tau}. \quad (10)$$

On the other hand, equation (9) can not be solved exactly. Since a general solution is being sought at a certain channel depth with τ and ξ as the independent variables, the resulting differential equation (9) can be replaced by a system of differential equations with a smaller number of independent variables. So we employ the line method, discussed in detail by Mikhlin [2], which lies midway between analytical and grid methods. The basis of the method is substitution of finite differences for the derivatives with respect to the independent η-variable. This approach replaces equation (9) by a system of differential equations with discontinuous coefficients and with only one independent variable ξ.

The finite difference quantities in the η-direction are

$$\left.\frac{\partial\phi}{\partial\eta}\right|_{\eta=\eta_k} = \frac{1}{h}[\phi_{k+1}(\xi) - \phi_k(\xi)]$$

and

$$\left.\frac{\partial^2\phi}{\partial\eta^2}\right|_{\eta=\eta_k} = \frac{1}{h^2}[\phi_{k+1}(\xi) - 2\phi_k(\xi) + \phi_{k-1}(\xi)]. \quad (11)$$

With the aid of equations (11), equation (9) becomes the following system of differential equations:

$$\psi_k\frac{d^2\phi}{d\xi_k^2} - \mu\chi_k\frac{d\phi}{d\xi_k} + \left(\frac{\psi}{h^2} + \frac{\psi'}{h}\right)_k\phi_{k+1} + \left(\lambda^2 - \frac{2\psi}{h^2} - \frac{\psi'}{h}\right)_k\phi_k + \frac{\psi_k}{h^2}\phi_{k-1} = 0 \quad (12)$$

where $k = 1, 2, \ldots, n$. The boundary conditions become $\phi_0(\xi) = \phi_1(\xi)$ and

$$\phi_{n+1}(\xi) = \phi_n(\xi). \quad (13)$$

The system of equations (12) and (13) contain n equations and n unknowns $\phi_1(\xi), \phi_2(\xi), \ldots, \phi_n(\xi)$. The coupled system of equations (12) and (13) can be uncoupled and written as

$$(g_1 r^{n+1} + g_2 r^n + \ldots + g_{n+1}r + g_{n+2})\,\phi_k = 0 \quad (14)$$

where $r = d/d\xi$ and the coefficients, g's, are functions of

$$\psi(\eta)_k, \quad \mu\chi(\eta)_k, \quad \psi'(\eta)_k, \quad h \text{ and } \lambda^2,$$

We shall now consider the above differential system where the coefficients in the equation (14) depend upon the eigenvalues λ^2 to be solutions to a positive definite eigenvalue problem. Since the concentration profiles are bounded, and differentiable, the assumptions of real and positive eigenvalues are valid. Sangren [3] gives a discussion of this issue. Since $C_{k\lambda}(\tau, \xi)$ is bounded, only the positive eigenvalues, $\lambda^2 > 0$, contribute to the solution of equation (14). Assuming that the polynomial in equation (14) has $n + 1$ imaginary roots of the type

$$r_n = p_n \pm iq_n \quad (15)$$

the solution for $\phi_k(\xi)$ is

$$\phi_{k\lambda}(\xi) = \sum_{i=1}^{n+1/2} e^{p_i\xi}(H_{2i\lambda}\cos q_i\xi + H_{3i\lambda}\sin q_i\xi) \quad (16)$$

where p_i and q_i are functions of λ which can be represented as

$$p_i = \sum_{j=0}^{\infty} a_{1j}\lambda^j + \frac{a_{2j}}{\lambda^j}$$

and

$$q_i = \sum_{j=0}^{\infty} b_{1j}\lambda^j + \frac{b_{2j}}{\lambda^j}. \tag{17}$$

The general solution for $C_{k\lambda}(\tau, \xi)$ is then

$$C_{k\lambda}(\tau,\xi) = \sum_{i=1}^{n+1/2} \exp\left\{\sum_{j=0}^{\infty}[a_{1j}\lambda^j + (a_{2j}/\lambda^j)]\xi - \lambda^2\tau\right\} \times \left\{A_{i\lambda}\cos\left[\sum_{j=0}^{\infty}(b_{1j}\lambda^j + b_{2j}/\lambda^j)\xi\right] + B_{i\lambda}\sin\left[\sum_{j=0}^{\infty}(b_{1j}\lambda^j + b_{2j}/\lambda^j)\xi\right]\right\}. \tag{18}$$

We can superimpose solutions for all the values of λ, since they are real and positive. Thus

$$C_k(\tau,\xi) = \int_0^{\infty} C_{k\lambda}(\tau,\xi)\,\mathrm{d}\lambda. \tag{19}$$

The integrals in equation (19) can be represented by Hermite polynomials which are orthogonal polynomials associated with a normal distribution. Equation (19) can be expanded and rearranged so that the integrals are of the following form:

$$He_n(\xi) = \mathrm{e}^{\xi^2}\frac{2^{n+1}}{\sqrt{\pi}}\int_0^{\infty}\mathrm{e}^{-\lambda^2}\lambda^n\,[\cos 2\xi\lambda\cos(n\pi/2) + \sin 2\xi\lambda\sin(n\pi/2)]\,\mathrm{d}\lambda. \tag{20}$$

This form of the Hermite polynomial integrals can be seen even in equation (19) where the exponential, the sine, and the cosine functions appear naturally as a result of the line integral technique. Equation (19) can be expanded and written as

$$C_k(\tau,\xi) = \int_0^{\infty}\sum_{i=1}^{n+1/2}\left(1 + a_{10}\xi + \frac{a_{10}^2\xi^2}{2!} + \ldots\right)\left(1 + a_{11}\lambda\xi + \frac{a_{11}^2\lambda^2\xi^2}{2!} + \ldots\right)\ldots\left(1 - \lambda^2\tau + \frac{\lambda^4\tau^2}{2!}\ldots\right)$$
$$\times\{A_{i\lambda}\cos(2\lambda\xi)[1 - (1/2!)(b_{10}\xi + b_{12}\lambda^2\xi + \ldots)^2 + (1/4!)(b_{10}\xi + b_{12}\lambda^2\xi + \ldots)^4\ldots] + B_{i\lambda}\sin(2\lambda\xi)$$
$$\times[b_{10}\xi + b_{12}\lambda^2\xi + \ldots) - (1/3!)(b_{10}\xi + b_{12}\lambda^2\xi + \ldots)^3]\}\,\mathrm{d}\lambda. \tag{21}$$

The contribution from each solution is superimposed, namely

$$\sum_{i=1}^{n+1/2} A_{i\lambda} = A(\lambda) = A_0 + A_1\lambda + A_2\lambda^2 + \ldots,$$

to give the following form of equation (26).

$$C_k(\tau,\xi) = \int_0^{\infty}\left(1 - \lambda^2\tau + \frac{\lambda^4\tau^2}{2!} - \ldots\right)\times\left(1 + a_{10}\xi + \frac{a_{10}^2\xi^2}{2!} + \ldots\right)\left(1 + a_{11}\lambda\xi + \frac{a_{11}^2\lambda^2\xi^2}{2!} + \ldots\right)\ldots\{(A_0 + A_1\lambda + A_2\lambda^2 + \ldots)$$
$$\times[1 - (1/2!)(b_{10}\xi + b_{12}\lambda^2\xi + \ldots)^2 + (1/4!)(b_{10}\xi + b_{12}\lambda^2\xi + \ldots)^4\ldots]\cos(2\lambda\xi)$$
$$+ (B_0 + B_1\lambda + B_2\lambda^2 + \ldots)[(b_{10}\xi + b_{12}\lambda^2\xi + \ldots) - (1/3!)(b_{10}\xi + b_{12}\lambda^2\xi + \ldots)^3 + \ldots]\sin(2\lambda\xi)\}\,\mathrm{d}\lambda. \tag{22}$$

Each term of the above integrals contributes to a certain order of Hermite polynomials. For example, the fourth order Hermite polynomial is obtained from the following terms:

$$\int_0^{\infty}\beta_4(\tau)[\lambda^4 - \lambda^6 + \lambda^8 - \lambda^{10} + \ldots + \lambda^4\xi^2 - \lambda^6\xi^2 + \lambda^8\xi^2 - \lambda^{10}\xi^2 + \ldots + \lambda^4\xi^4 - \lambda^6\xi^4 + \lambda^8\xi^4 - \lambda^{10}\xi^4 + \ldots]\cos(2\lambda\xi)\,\mathrm{d}\lambda.$$

These integrals in equation (22) can be written in the form of a series for the kth strip of the channel,

$$C_k(\tau,\xi) = C_N(\tau,\xi)\left[1 + \sum_{j=3}^{\infty}\beta_j(\tau)He_j(\xi)\right], \tag{23}$$

or as the strip size $h \to 0$, equation (23) is written as

$$C(\tau,\eta,\xi) = C_N(\tau,\eta,\xi)[1 + \sum_{j=3}^{\infty}\beta_j(\tau,\eta)He_j(\xi)] \tag{24}$$

where

$$C_N(\tau,\eta,\xi) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{\xi^2}{2\sigma^2}\right)$$

and σ^2 is the variance of the longitudinal concentration distribution. This representation, equation (24), is not a unique, but a convenient one.

Each term of the Hermite polynomial series in equation (23) applies corrections to the long-term asymptotic Gaussian solution. $\beta_3(\tau)He_3(\xi)$ contributes corrections due to the skewness factor, and $\beta_4(\tau)He_4(\xi)$ contributes corrections due to the flatness factor, etc. The time dependent series coefficients, $\beta_3(\tau)$, $\beta_4(\tau), \ldots, \beta_n(\tau)$ can be evaluated by employing Aris moment solutions. Consider first a Hermite polynomial series up to the fourth moment.

$$C(\tau,\xi) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{\xi^2}{2\sigma^2}\right)[1 + \beta_3(\tau)He_3(\xi) + \beta_4(\tau)He_4(\xi)] \tag{25}$$

where $He_3(\xi) = 8\xi^3 - 12\xi$ and $He_4(\xi) = 16\xi^4 - 48\xi^2 + 12$. The third and the fourth moments can be calculated from equation (25).

$$C_3(\tau) = \int_{-\infty}^{\infty}\xi^3 C(\tau,\xi)\,\mathrm{d}\xi \tag{26}$$

and

$$C_4(\tau) = \int_{-\infty}^{\infty} \xi^4 C(\tau, \xi)\, d\xi. \qquad (27)$$

The result of equation (26) gives $\beta_3(\tau)$, the skewness contributor.

$$\beta_3(\tau) = C_3(\tau)/[\sigma^4(120\sigma^2 - 36)].$$

Similarly the result of equation (27) gives $\beta_4(\tau)$, the flatness contributor.

$$\beta_4(\tau) = [C_4(\tau) - 3\sigma^4]/[\sigma^4(1680\,\sigma^4 - 720\,\sigma^2 + 36)] \qquad (29)$$

The behaviors of $\beta_3(\tau)$ and $\beta_4(\tau)$ as functions of time are shown in Fig. 1. Notice that the contribution from each series term approaches zero as $\tau \to \infty$; this validates the extraction of the normal distribution from equation (23).

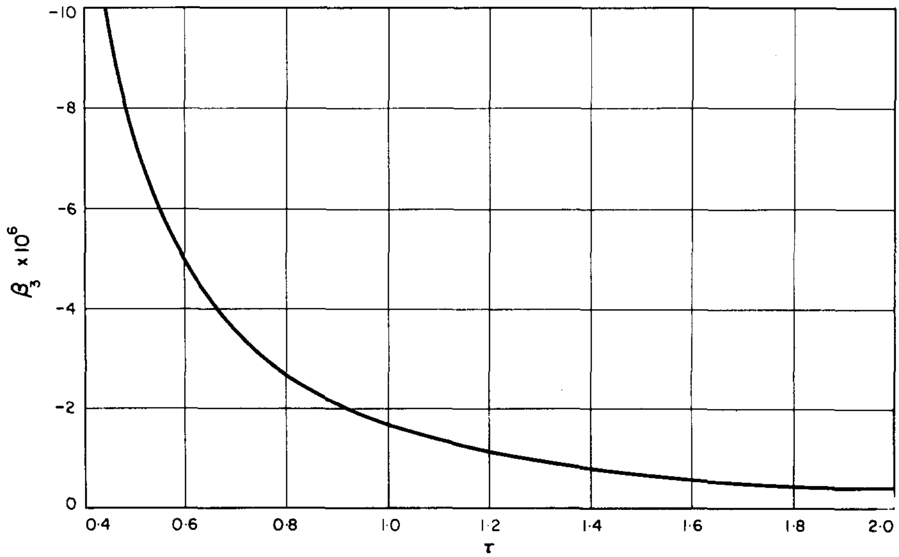

FIG. 1a. Coefficient of the skewness factor in Hermite polynomial series.

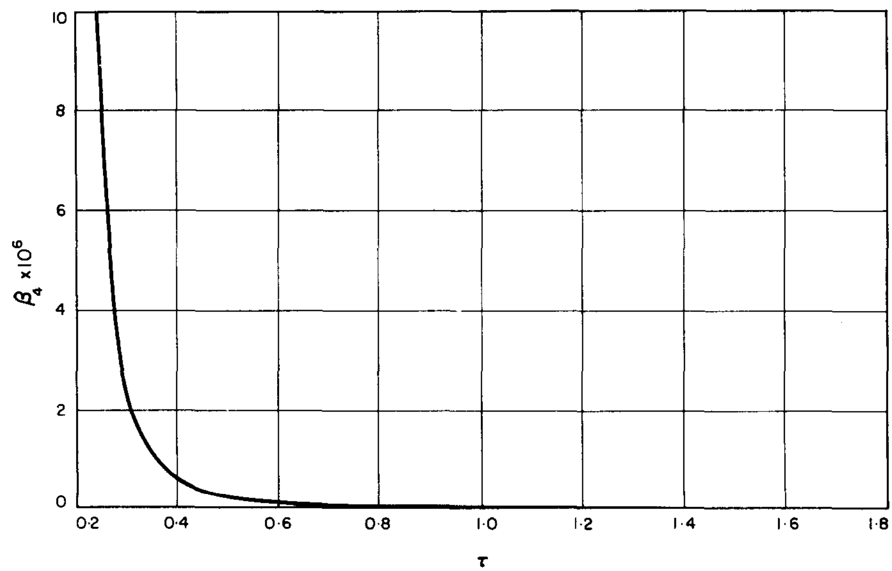

FIG. 1b. Coefficient of the flatness factor in Hermite polynomial series.

Equation (23) is a simple series representation for the longitudinal concentration distribution where each term represents a correction to the normal distribution due to skewness, flatness, etc. There are two practical criteria that must be satisfied in a computation of this type. The computed Hermite polynomial coefficients, β_j's, should decrease fairly rapidly in magnitude in time and successive approximations should appear to converge to a common value. Inspection of Fig. 1 shows that the first criterion is satisfied. Inspections of Table 1 and Fig. 2 show that the second criterion is satisfied. Extensive comparisons with the experimental data of Sayre [4, 5] show in Fig. 3 that only moments up to the flatness factor are required for accurate predictions. That is, only three terms of the series of equation (23) are needed.

The functional form of the concentration distribution proposed here provides a convenient computation method which should assist in further application of the Aris moment method in the solution of the diffusion equation.

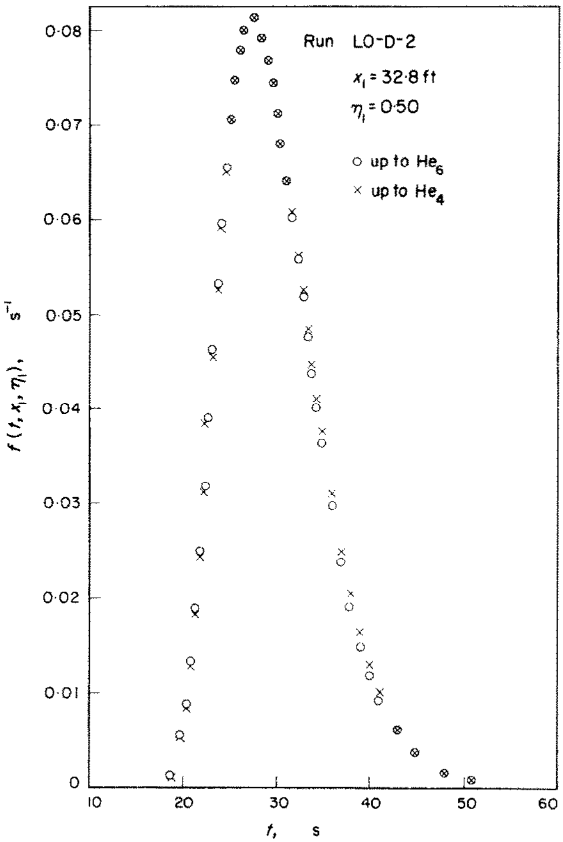

FIG. 2. Corrections due to the higher terms in Hermite polynomial series.

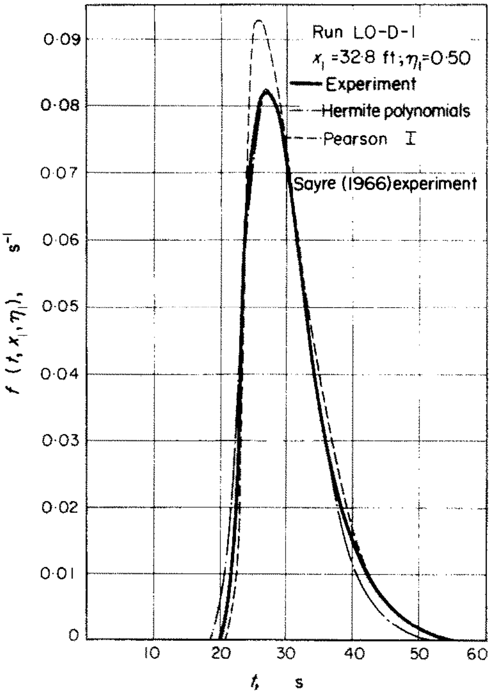

FIG. 3a. Comparison of experimental and theoretical longitudinal concentration curves.

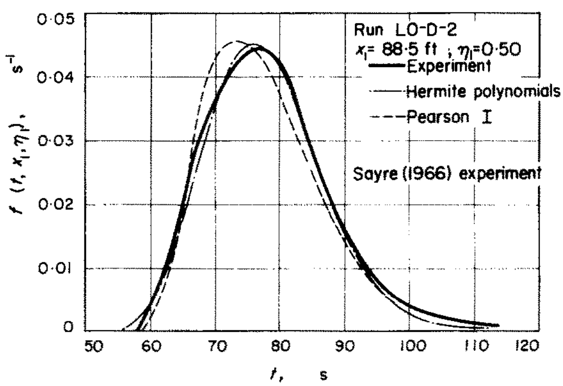

FIG. 3b. Comparison of experimental and theoretical longitudinal concentration curves.

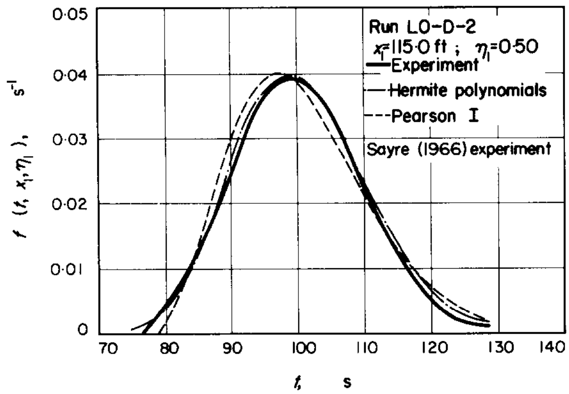

FIG. 3c. Comparison of experimental and theoretical longitudinal concentration curves.

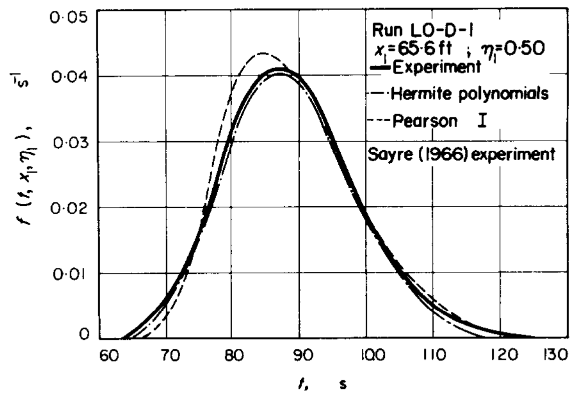

FIG. 3d. Comparison of experimental and theoretical longitudinal concentration curves.

Table 1. Prediction of Sayre's (10) LO-D-2 experiment at x = 32·8 *ft with the present Hermite polynimial series*

t(s)	Normal distribution $C_N(\tau, \xi) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{\xi^2}{2\sigma^2}\right)$	Skewness $C = C_N(1+\beta_3 He_3)$	Flatness $C = C_N(1+\beta_3 He_3 + \beta_4 He_4)$	$C_N\beta_3 He_3$	$C_N\beta_4 He_4$	$\frac{\beta_3 He_3}{\beta_4 He_4}$
19	0·00432	0·00151	0·00119	−0·00280	−0·00032	9
20	0·00955	0·00566	0·00527	−0·00389	−0·00039	10
21	0·01787	0·01349	0·01311	−0·00438	−0·00038	12
22	0·02895	0·02487	0·02456	−0·00408	−0·00031	13
23	0·04178	0·03863	0·03843	−0·00315	−0·00020	16
24	0·05482	0·05283	0·05273	−0·00199	−0·00010	20
25	0·06636	0·06537	0·06534	−0·00099	−0·00003	33
26	0·07509	0·07475	0·07475	−0·00034	−0·00000	large
27	0·08015	0·08010	0·08010	−0·00005	−0·00000	large
28	0·08148	0·08149	0·08149	0·00001	−0·00000	large
29	0·07938	0·07937	0·07937	−0·00001	−0·00000	large
30	0·07446	0·07451	0·07451	0·00005	−0·00000	large
31	0·06766	0·06791	0·06791	0·00025	−0·00000	large
32	0·05975	0·06033	0·06031	0·00058	−0·00002	29
33	0·05144	0·05244	0·05239	0·00100	−0·00005	20
34	0·04332	0·04475	0·04468	0·00143	−0·00007	20
35	0·03576	0·03758	0·03748	0·00182	−0·00010	18
36	0·02901	0·03113	0·03099	0·00212	−0·00014	15
37	0·02315	0·02546	0·02529	0·00231	−0·00017	14
38	0·01821	0·02059	0·02040	0·00238	−0·00019	13
39	0·01415	0·01650	0·01630	0·00235	−0·00020	12
40	0·01085	0·01309	0·01289	0·00224	−0·00020	11
41	0·00823	0·01030	0·01010	0·00207	−0·00020	10
42	0·00620	0·00805	0·00786	0·00185	−0·00019	10
43	0·00461	0·00623	0·00606	0·00162	−0·00017	9
44	0·00340	0·00479	0·00464	0·00139	−0·00015	9
45	0·00250	0·00367	0·00353	0·00117	−0·00014	8
46	0·00182	0·00279	0·00267	0·00097	−0·00012	8
47	0·00132	0·00210	0·00200	0·00078	−0·00010	8
48	0·00095	0·00158	0·00150	0·00063	−0·00008	8
49	0·00068	0·00118	0·00111	0·00050	−0·00007	7
50	0·00048	0·00088	0·00082	0·00040	−0·00006	7

The contributions from higher terms of the Hermitynomials get less as shown in Fig. 2.

REFERENCES

1. R. ARIS, On the dispersion of a solute in a fluid flowing through a tube, *Proc. R. Soc. Lond.*, **235A**, 66-77 (1956).
2. S. G. MIKHLIN and K. L. SMOLITSKIY, *Approximate Methods for Solution of Differential and Integral Equations*. American Elsevier, New York (1967).
3. W. C. SANGREN, Differential equations with discontinuous coefficients, Oak Ridge Nat. Lab. Rep no. 1566 Oak Ridge, Tenn. (1953).
4. W. W. SAYRE, Dispersion of mass in open channel flow, Ph.D. Thesis submitted at Colorado State University (1967).
5. W. W. SAYRE, Dispersion of silt particles in open channel flow, *J. Hyd. Div. Proc. ASCE* **95** (HY3), 1009–1038 (1969).

Int. J. Heat Mass Transfer. Vol. 14, pp. 2152–2156. Pergamon Press 1971. Printed in Great Britain

BUBBLE CONDENSATION WITH NON-HOMOGENEOUS DISTRIBUTION OF NON-CONDENSABLES

DAVID MOALEM and SAMUEL SIDEMAN*

Department of Chemical Engineering, Technion Israel Institute of Technology, Haifa, Israel

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NOMENCLATURE

C_p, specific heat;
F_1, $F_1(\beta_f^H)$, function of β_f^H, equation (20);
F_2, $F_2(\beta_f^P)$, function of β_f^P, equation (24);
G^*, ratio, liquid to vapor density, $\rho_L/\rho_v{}^*$;
Ja, Jacob number $(\rho C_p \Delta T/\lambda\rho_v)$;
k, thermal conductivity;
K_v, velocity factor, modified potential flow;
P^*, total system pressure;
P_g, partial pressure, non condensible;
P_v, partial pressure vapor;
Pe, Péclét number $(U_\infty 2R/\alpha)$;
Pr, Prandtl number $(\mu C_p/k)$;
Nu, Nusselt number $(h2R/k)$;
q, heat flux [Btu/hft^2];
q_0, heat flux, potential flow [Btu/hft^2];
R, radius of bubble;
R_0, initial radius of bubble;
R_f, final radius of bubble;
$\hat{R}$, specific gas constant;
r, radial coordinate;
T^*, saturation temperature corresponding to P^*;
T_w, bubble wall temperature;
T_∞, approach temperature, surrounding liquid;
ΔT, temperature difference, $T^* - T_\infty$;
t, time;
U, velocity of rise;
α, thermal diffusivity;
β, dimensionless radius, R/R_0;
β_f, final dimensionless radius, R_f/R_0;
λ, latent heat;
ρ, density, continuous phase;
ρ_L, density, condensate;
ρ_v, density, vapor;
τ, dimensionless time, Fourier number $(=\alpha t/R_0^2)$;
$\hat{\tau}$, dimensionless time for collapsing bubble $(JaPe^{\frac{1}{2}}\tau)$.

Subscripts
f, final;
0, initial;
w, at the wall.

Superscripts
H, homogeneous distribution;
P, parabolic distribution.

INTRODUCTION

WITTKE and Chao [1] and Isenberg and Sideman [2] presented numerical solutions for unsteady state bubble collapse; the former for a single component (steam–water) system and the latter for a two component (pentane–water) system. These systems differ since the condensate in a single component bubble merges with the surrounding liquid, while the condensate in the two component system remains within the confines of the bubble walls. More recently, Sideman *et al.* [3] presented an approximate, quasi-steady state, analytical solution for bubble collapse in two-component, 3-phase systems. The solution is general, conveniently reducing to a solution for a single component

* Presently: Visiting Professor, Department of Chemical Engineering, University of Houston, Houston, Texas 77004.